

ELECTROWEAK SUPERSYMMETRIC QUANTUM CORRECTIONS TO THE TOP QUARK WIDTH

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ABSTRACT

Within the framework of the MSSM, we compute the electroweak one-loop supersymmetric quantum corrections to the width $\Gamma(t \rightarrow W^+ b)$ of the canonical main decay of the top quark. The results are presented in two on-shell renormalization schemes parametrized either by α or G_F . While in the standard model, and in the Higgs sector of the MSSM, the electroweak radiative corrections in the G_F -scheme are rather insensitive to the top quark mass and are of order of 1% at most, the rest (“genuine” part) of the supersymmetric quantum effects in the MSSM amount to a non-negligible correction that could be about one order of magnitude larger, depending on the top quark mass and of the region of the supersymmetric parameter space. These new electroweak effects, therefore, could be of the same order (and go in the same direction) as the conventional leading QCD corrections.

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1. Introduction

The top quark and the Higgs boson share the privilege of being the last two building blocks that remain to be found experimentally to confirm the fundamental spectrum of the Standard Model (SM) [1], and as such the theoretical consistency of the model heavily hinges on the existence of these two particles. The replication of the doublet/singlet pattern structure of the first two fermion families is required for the suppression of the FCNC in B -meson decays [2]. Moreover, there is a lot of indirect experimental evidence on the existence of the weak isospin partner of the bottom quark. The isospin quantum numbers of the b -quark can be directly measured through the partial Z -decay width to $b\bar{b}$ pairs and the forward-backward asymmetry of b -quarks at the Z -peak yielding, within small error bars, $T^3(b_L) = -0.5$ [3]. Similarly, although with much lesser accuracy, the isospin of the RH-component is compatible with zero [3].

In spite of being a sequential fermion, the top quark plays a special role in the fermion families due to its huge mass m_t . Primordially, the SM predicts a comparatively strong direct interaction with the Higgs sector through a large Yukawa coupling [4]. We thus may expect the top quark as a particularly helpful laboratory for testing the symmetry breaking mechanism of the SM. It may also help to unravel physical effects beyond the SM, such as e.g. those predicted by the Minimal Supersymmetric Standard Model (MSSM) [5]. Direct searches at Tevatron, put a limit of $m_t > 113 \text{ GeV}$ [6], whereas the combined electroweak data fits from LEP (in the pure context of the SM) predict ⁴ [8]

$$m_t = 166 \pm 18 \pm 21 \text{ GeV}, \quad (1)$$

where the first error is due to measurement errors, while the second arises from the uncertainty in the Higgs mass, taken to be between 60 GeV and 1 TeV . Thus, all phenomenological evidence points towards the top quark being around the corner, and may be within the discovery potential of the Tevatron ($m_t \leq 180 \text{ GeV}$). At the hadron-hadron supercollider LHC and a next linear e^+e^- collider, the $t\bar{t}$ system will be copiously produced through parton fusion and e^+e^- annihilation, respectively, and the decay modes analyzed in great detail [9, 10]. Therefore, precise measurements of the top quark properties will become available facing the predictions of the SM, and we should be prepared to recognize or to exclude hints of new physics. Notice that for $m_t \geq 130 \text{ GeV}$, the width $\Gamma_t \equiv \Gamma(t \rightarrow W^+ b)$ exceeds $\sim 0.5 \text{ GeV}$ and thus $\Gamma_t > \Lambda_{QCD}$. As a consequence, the top quark will predominantly decay as a free quark, and the bound states cannot be formed, leading to a broad threshold enhancement in the production process of $t\bar{t}$ pairs instead

⁴From the analysis of Δr in the MSSM, it follows that m_t could be $\sim 20 - 30 \text{ GeV}$ lighter than expected from the SM, as shown in ref.[7]

of sharp resonances [11]. This allows to analyze the production and decay of top quarks perturbatively, with Γ_t serving as the infrared cutoff [12].

Radiative corrections to conventional physical processes [13] are a powerful tool to search for mass scales within and beyond the SM, and they offer us the opportunity to peep at sectors of the theory that are not (yet) directly observable. In this paper we concentrate on the computation of the supersymmetric (SUSY) quantum effects on the width of the canonical decay $t \rightarrow W^+ b$, probably the main decay mode of the top quark. In extended versions of the SM (Cf.sect.4) other decay channels may also be open, but the standard decay always has a sizeable branching ratio. In the framework of the SM, the aforementioned limit on the top quark mass is based on that standard decay and the tagging of the subsequent leptonic decay mode of the weak gauge boson, with an approximate branching ratio of $BR(t \rightarrow l \nu_l b) \simeq 1/9$. Detailed analyses of the electroweak one-loop effects on the canonical decay, in the pure context of the SM, already exist in the literature, with the result (somewhat surprising) that they are of order of $1 - 1.5\%$ at most and they turn out to be rather insensitive to the top quark mass in the relevant range $100 - 200 \text{ GeV}$ [14, 15, 16].

The motivation of this calculation are the potentially large quantum effects on the top decay width arising from extra significant interactions between heavy fermions and the Higgs sector. In supersymmetric extensions of the SM, the Higgs sector contains at least two superfield Higgs doublets. The corresponding analysis for two-doublet Higgs (SUSY and non-SUSY) extensions of the SM was first given in ref.[17], with the result that no large corrections ($\leq 1\%$) on the top quark width are gained from the SUSY scalar Higgses alone. Notwithstanding, a full account of the remaining –“genuine”– SUSY contribution: namely, from sfermions (squarks and sleptons) and “inos” (charginos and neutralinos), was still missing, but the relevant Higgs-like interactions involving these two set of supersymmetric particles (“sparticles”) provide another source of large loop contributions, in particular if the sparticles are not too heavy. Such an interesting situation of a “light” effective low energy SUSY scale M_{SUSY} (i.e. that scale fixed by the renormalized soft SUSY-breaking terms) around the Fermi scale (or even below) is compatible with the intriguing coupling constant unification in a SUSY-GUT scenario [18] consistent with the high precision LEP data and the non-observation of proton decay [19, 20]. In this paper we exploit the possibility to obtain indirect information both on SUSY physics and on top quark dynamics.

The paper is organized as follows. In sect.2 we present a quick review of the basic SUSY formalism necessary for our calculation and give those parts of the interaction Lagrangian describing the fermion-sfermion-chargino/neutralino coupling. In sect.3 we display the results of the analytical calculation of the various electroweak one-loop MSSM

contributions to the top quark decay width within the framework of the standard on-shell renormalization framework. Finally, sect.4 is devoted to a detailed presentation of the numerical analysis and a corresponding discussion of the results.

2. SUSY Formalism and Interaction Lagrangian

We shall perform our calculations in a mass-eigenstate basis. One goes from the weak-eigenstate basis to the mass-eigenstate basis via appropriate unitary transformations. Two classes of SUSY particles enter our calculations: the fermionic partners of gauge bosons and Higgs bosons (called gauginos, \tilde{W} , and higgsinos, \tilde{H} , respectively) and on the other hand the scalar partners of quarks and leptons (called squarks, \tilde{q} , and sleptons, \tilde{l} , respectively, or sfermions, \tilde{f} , generically). Within the context of the MSSM, we need two Higgs superfield doublets with weak hypercharges $Y_{1,2} = \mp 1$ (hats denote superfields):

$$\hat{H}_1 = \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^- \end{pmatrix} \quad , \quad \hat{H}_2 = \begin{pmatrix} \hat{H}_2^+ \\ \hat{H}_2^0 \end{pmatrix} . \quad (2)$$

The corresponding scalar Higgs doublet H_1 (H_2) gives mass to the down (up) -like quarks through the VEV $\langle H_1^0 \rangle = v_1$ ($\langle H_2^0 \rangle = v_2$). This is seen from the structure of the MSSM superpotential [5]

$$\hat{W} = \epsilon_{ij} [h_b \hat{H}_1^i \hat{Q}^j \hat{D} + h_t \hat{H}_2^j \hat{Q}^i \hat{U} + \mu \hat{H}_1^j \hat{H}_2^i] , \quad (3)$$

where we have singled out only the Yukawa couplings of the third quark generation, (t, b) , as a generical fermion-sfermion generation of chiral matter superfields \hat{Q} , \hat{U} and \hat{D} . Their respective scalar (squark) components are:

$$\tilde{Q} = \begin{pmatrix} \tilde{t}'_L \\ \tilde{b}'_L \end{pmatrix} \quad , \quad \tilde{U} = \tilde{t}_R^* \quad , \quad \tilde{D} = \tilde{b}_R^* , \quad (4)$$

with weak hypercharges $Y_Q = +1/3$, $Y_U = -4/3$ and $Y_D = +2/3$. The primes in (4) denote the fact that $\tilde{q}'_a = \{\tilde{q}'_L, \tilde{q}'_R\}$ are weak-eigenstates, not mass-eigenstates. However, there may be “chiral” L-R mixing between weak-eigenstate sfermions of a given flavor (except for the sneutrinos), which is induced already at tree-level by the μ -term in the superpotential and by the (renormalized) trilinear “soft” SUSY-breaking terms [5]. Due to this mixing, we have to derive the corresponding squark mass-eigenstates $\tilde{q}_a = \{\tilde{q}_1, \tilde{q}_2\}$ by means of appropriate 2×2 rotation matrices, $R^{(q)}$, that diagonalize the chiral mass matrices (we neglect intergenerational mixing):

$$\tilde{q}_a = \sum_b R_{ab}^{(q)} \tilde{q}'_b \quad (\tilde{q} = \tilde{t}, \tilde{b}). \quad (5)$$

From the higgsinos and the various gauginos we form the following three sets of two-component Weyl spinors:

$$\Gamma_i^+ = \{-i\tilde{W}^+, \tilde{H}_2^+\} , \quad \Gamma_i^- = \{-i\tilde{W}^-, \tilde{H}_1^-\} , \quad (6)$$

$$\Gamma_\alpha^0 = \{-i\tilde{B}^0, -i\tilde{W}_3^0, \tilde{H}_2^0, \tilde{H}_1^0\} . \quad (7)$$

These states get mixed up when the neutral Higgs fields acquire nonvanishing VEV's giving masses to the gauge bosons: $M_W^2 = (1/2)g^2(v_1^2 + v_2^2)$, $M_Z^2 = (1/2)(g^2 + g'^2)(v_1^2 + v_2^2)$. The “ino” mass Lagrangian reads

$$\mathcal{L}_M = - < \Gamma^+ | \mathcal{M} | \Gamma^- > - \frac{1}{2} < \Gamma^0 | \mathcal{M}^0 | \Gamma^0 > + h.c. , \quad (8)$$

where the charged and neutral gaugino-higgsino mass matrices are the following:

$$\mathcal{M} = \begin{pmatrix} M & M_W \sqrt{2} c_\beta \\ M_W \sqrt{2} s_\beta & \mu \end{pmatrix} \quad (9)$$

and

$$\mathcal{M}^0 = \begin{pmatrix} M' & 0 & M_Z s_\beta s_\theta & -M_Z c_\beta s_\theta \\ 0 & M & -M_Z s_\beta c_\theta & -M_Z c_\beta c_\theta \\ M_Z s_\beta s_\theta & -M_Z s_\beta c_\theta & 0 & -\mu \\ -M_Z c_\beta s_\theta & M_Z c_\beta c_\theta & -\mu & 0 \end{pmatrix} , \quad (10)$$

with the following notation:

$$s_\beta \equiv \sin \beta \quad , \quad c_\beta \equiv \cos \beta \quad , \quad \tan \beta = \frac{v_2}{v_1} \quad , \quad c_\theta \equiv M_W/M_Z \quad , \quad s_\theta^2 \equiv 1 - c_\theta^2 . \quad (11)$$

The mass parameters M and M' come from $SU(2)_L \times U(1)_Y$ -invariant gaugino mass terms that softly break global SUSY, while μ is the very same SUSY Higgs mass term in the superpotential (3). We shall assume that the MSSM can be embedded in a GUT, in which case the parameters M' and M are related as follows [5, 18]

$$\frac{M'}{M} = \frac{5}{3} t_\theta^2 \simeq 0.5 , \quad (12)$$

where $t_\theta \equiv s_\theta/c_\theta$. The 2×2 mass matrix \mathcal{M} is in general non-symmetrical and its diagonalization is accomplished by two unitary matrices U and V , whereas the symmetrical 4×4 mass matrix \mathcal{M}^0 can be diagonalized by a single unitary matrix N :

$$U^* \mathcal{M} V^\dagger = \text{diag}\{M_1, M_2\} \quad , \quad N^* \mathcal{M}^0 N^\dagger = \text{diag}\{M_1^0, \dots, M_4^0\} . \quad (13)$$

Let us now build the charged mass-eigenstate 4-spinors (charginos) associated to the mass eigenvalues M_i . Call them Ψ_i^+ , and let Ψ_i^- be the corresponding charge conjugate states. We have⁵

$$\Psi_i^+ = \begin{pmatrix} U_{ij} \Gamma_j^+ \\ V_{ij}^* \bar{\Gamma}_j^- \end{pmatrix} \quad , \quad \Psi_i^- = C \bar{\Psi}_i^{-T} = \begin{pmatrix} V_{ij} \Gamma_j^- \\ U_{ij}^* \bar{\Gamma}_j^+ \end{pmatrix} . \quad (14)$$

⁵We use the notation of ref.[21] for the sparticles and their indices. We remark that first Latin indices a,b,...=1,2 are reserved for sfermions, middle Latin indices i,j,...=1,2 for charginos, and first Greek indices $\alpha, \beta, \dots = 1, 2, \dots, 4$ for neutralinos.

As for the neutral mass-eigenstate 4-spinors (neutralinos) associated to the mass eigenvalues M_α^0 , they are the following Majorana spinors

$$\Psi_\alpha^0 = \begin{pmatrix} N_{\alpha,\beta} \Gamma_\beta^0 \\ N_{\alpha,\beta}^* \bar{\Gamma}_\beta^0 \end{pmatrix} = C \bar{\Psi}_\alpha^{0T} . \quad (15)$$

The process-dependent SUSY diagrams contributing to $t \rightarrow W^+ b$ include only a limited portion of the MSSM Lagrangian. On the other hand, the computation of the (universal) counterterms (Cf. sect.3) associated to the on-shell renormalization procedure does require the use of the full electroweak SUSY part. We refer the reader to the literature for the remaining structure of the MSSM Lagrangian [5, 18, 21]. Here, however, we shall exhibit explicitly only that part of the interaction Lagrangian needed for the computation of the specific one-loop vertices related to our process, emphasizing that part with the relevant Yukawa couplings in the mass-eigenstate basis. In order to construct it, we project the quark-squark-higgsino terms from the superpotential (3). Furthermore, there are also gaugino interactions that mix with these terms; they come from expanding the SUSY counterpart of the $SU(2)_L \times U(1)_Y$ fermion-gauge interaction terms, viz.

$$\mathcal{L}_{\tilde{q}\lambda q} = i\sqrt{2} g_r \tilde{q}^* T^r \lambda^r q + h.c. , \quad (16)$$

with

$$\lambda^r = \{\vec{W}, \vec{B}\} \quad , \quad T^r = \{\vec{\sigma} / 2, Y/2\} \quad , \quad g_r = g, g' . \quad (17)$$

After adding up these two kinds of terms in the weak-eigenstate, two-component, basis and re-expressing the result in four-component notation and in the mass-eigenstate basis we find

$$\begin{aligned} \mathcal{L}_{\Psi q \tilde{q}} = & \sum_{a=1,2} \sum_{i=1,2} \left\{ -g \tilde{t}_a^* \bar{\Psi}_i^- \frac{1}{2} (A_{ai}^{(t)} - B_{ai}^{(t)} \gamma_5) b - g \tilde{b}_a^* \bar{\Psi}_i^+ \frac{1}{2} (A_{ai}^{(b)} - B_{ai}^{(b)} \gamma_5) t \right\} \\ & \sum_{a=1,2} \sum_{\alpha=1,\dots,4} \left\{ -\frac{g}{\sqrt{2}} \tilde{t}_a^* \bar{\Psi}_\alpha^0 \frac{1}{2} (A_{a\alpha}^{(t)} - B_{a\alpha}^{(t)} \gamma_5) t + \frac{g}{\sqrt{2}} \tilde{b}_a^* \bar{\Psi}_\alpha^0 \frac{1}{2} (A_{a\alpha}^{(b)} - B_{a\alpha}^{(b)} \gamma_5) b \right\} \\ & + h.c. , \end{aligned} \quad (18)$$

where, using the notation introduced above, we have defined the following coupling matrices:

$$\begin{aligned} A_{ai}^{(t)} &= R_{a1}^{(t)} (U_{i1}^* - \lambda_b V_{i2}) - \lambda_t R_{a2}^{(t)} U_{i2}^* , \\ B_{ai}^{(t)} &= R_{a1}^{(t)} (U_{i1}^* + \lambda_b V_{i2}) - \lambda_t R_{a2}^{(t)} U_{i2}^* , \end{aligned}$$

$$\begin{aligned} A_{ai}^{(b)} &= R_{a1}^{(b)} (V_{i1}^* - \lambda_t U_{i2}) - \lambda_b R_{a2}^{(b)} V_{i2}^* , \\ B_{ai}^{(b)} &= R_{a1}^{(b)} (V_{i1}^* + \lambda_t U_{i2}) - \lambda_b R_{a2}^{(b)} V_{i2}^* , \end{aligned}$$

$$\begin{aligned}
A_{a\alpha}^{(t)} &= R_{a1}^{(t)} (N_{\alpha 2}^* + \frac{1}{3} t_\theta N_{\alpha 1}^* + \sqrt{2} \lambda_t N_{\alpha 3}) - R_{a2}^{(t)} (\frac{4}{3} t_\theta N_{\alpha 1} - \sqrt{2} \lambda_t N_{\alpha 3}^*), \\
B_{a\alpha}^{(t)} &= R_{a1}^{(t)} (N_{\alpha 2}^* + \frac{1}{3} t_\theta N_{\alpha 1}^* - \sqrt{2} \lambda_t N_{\alpha 3}) + R_{a2}^{(t)} (\frac{4}{3} t_\theta N_{\alpha 1} + \sqrt{2} \lambda_t N_{\alpha 3}^*), \\
A_{a\alpha}^{(b)} &= R_{a1}^{(b)} (N_{\alpha 2}^* - \frac{1}{3} t_\theta N_{\alpha 1}^* - \sqrt{2} \lambda_b N_{\alpha 4}) - R_{a2}^{(b)} (\frac{2}{3} t_\theta N_{\alpha 1} + \sqrt{2} \lambda_b N_{\alpha 4}^*), \\
B_{a\alpha}^{(b)} &= R_{a1}^{(b)} (N_{\alpha 2}^* - \frac{1}{3} t_\theta N_{\alpha 1}^* + \sqrt{2} \lambda_b N_{\alpha 4}) + R_{a2}^{(b)} (\frac{2}{3} t_\theta N_{\alpha 1} - \sqrt{2} \lambda_b N_{\alpha 4}^*). \quad (19)
\end{aligned}$$

The potentially significant Yukawa couplings are contained in the following ratios with respect to the $SU(2)_L$ gauge coupling:

$$\lambda_t \equiv \frac{h_t}{g} = \frac{m_t}{\sqrt{2} M_W s_\beta} \quad , \quad \lambda_b \equiv \frac{h_b}{g} = \frac{m_b}{\sqrt{2} M_W c_\beta} . \quad (20)$$

Finally, the relevant charged-current interaction of squarks and charginos with the W^\pm gauge bosons is given by

$$\begin{aligned}
\mathcal{L}_W^{CC} &= \frac{ig}{\sqrt{2}} \sum_{a,c} \{ R_{a1}^{(b)*} R_{c1}^{(t)} \tilde{t}_c^* \overset{\leftrightarrow}{\partial}^\mu \tilde{b}_a W_\mu^+ \} + J_+^\mu W_\mu^- + h.c. \\
&+ \frac{1}{2} g^2 \sum_{a,c} \{ R_{a1}^{(t)*} R_{c1}^{(t)} \tilde{t}_c^* \tilde{t}_a + R_{a1}^{(b)*} R_{c1}^{(b)} \tilde{b}_c^* \tilde{b}_a \} W_\mu^+ W^{-\mu} , \quad (21)
\end{aligned}$$

where

$$J_+^\mu = g \sum_\alpha \sum_i \bar{\Psi}_\alpha^0 \gamma^\mu (C_{\alpha i}^L P_L + C_{\alpha i}^R P_R) \Psi_i^+ , \quad (22)$$

with $P_{L,R} = (1/2)(1 \pm \gamma_5)$, and the chargino coupling matrices

$$\begin{aligned}
C_{\alpha i}^L &= \frac{1}{\sqrt{2}} N_{\alpha 3} U_{i2}^* - N_{\alpha 2} U_{i1}^* \\
C_{\alpha i}^R &= \frac{-1}{\sqrt{2}} N_{\alpha 4}^* V_{i2} - N_{\alpha 2}^* V_{i1} . \quad (23)
\end{aligned}$$

3. Supersymmetric Quantum Corrections

In our calculation of the one-loop electroweak corrections to $\Gamma_t \equiv \Gamma(t \rightarrow W^+ b)$ in the MSSM, we shall adopt the on-shell renormalization scheme [22], where the fine structure constant, α , and the masses of the gauge bosons, fermions and scalars are the renormalized parameters: $(\alpha, M_W, M_Z, M_H, m_f, M_{SUSY}, \dots)$ ⁶. We will, for brevity sake, refer to it as

⁶ For a comprehensive review, see e.g. refs. [13, 23, 24].

the “ α -scheme”: (α, M_W, M_Z) . As stated, the corrections to Γ_t from a general two-Higgs-doublet model (2HDM), and in particular that of the MSSM, were already considered in ref.[17] within the framework of the “minimal” α -scheme of ref.[23]⁷, which we shall also adhere to in this work. We shall therefore concentrate on the remaining supersymmetric electroweak corrections: namely, from charginos, neutralinos and sfermions. The direct vertex corrections originating from this “genuine” supersymmetric sector of the MSSM are depicted in Fig.1. The bare structure of any of these vertices can be separated as the sum of the tree-level part plus one-loop correction:

$$\Gamma_\mu^{(0)} = i \frac{g}{\sqrt{2}} [\gamma_\mu P_L (1 + F_L) + \gamma_\mu P_R F_R + \frac{p_\mu}{M_W} (P_L H_L + P_R H_R)], \quad (24)$$

where the correction has been parametrized in terms of four form factors F_L, F_R, H_L and H_R , of which only F_L is UV-divergent [17]. The corresponding renormalized vertex $\Gamma^\mu \rightarrow \Gamma^\mu + \delta\Gamma^\mu$ is obtained from the renormalized Lagrangian $\mathcal{L} \rightarrow \mathcal{L} + \delta\mathcal{L}$ in the on-shell renormalization framework. In the minimal α -scheme of ref.[23], where a minimum number of field renormalization constants is used (viz. one renormalization constant per symmetry multiplet), the effect of the counterterm Lagrangian $\delta\mathcal{L}$ is equivalent to the following replacement of the UV-divergent form factor in eq.(24):

$$F_L \rightarrow F_L + \delta Z_1^W - \delta Z_2^W + \delta Z_L, \quad (25)$$

and the resulting expression has to be finite. Here $Z_i = 1 + \delta Z_i$ are the renormalization constants defined by [23]

$$\begin{aligned} W_\mu &\rightarrow (Z_2^W)^{1/2} W_\mu \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix} &\rightarrow Z_L^{1/2} \begin{pmatrix} t_L \\ b_L \end{pmatrix} \\ g &\rightarrow (Z_1^W) (Z_2^W)^{-3/2} g. \end{aligned} \quad (26)$$

Explicitly they read as follows

$$\begin{aligned} \delta Z_1^W &= \left. \frac{\Sigma^\gamma(k^2)}{k^2} \right|_{k^2=0} + \frac{1 + 2c_\theta^2}{s_\theta c_\theta} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_\theta^2}{s_\theta^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \\ \delta Z_2^W &= \left. \frac{\Sigma^\gamma(k^2)}{k^2} \right|_{k^2=0} + 2 \frac{c_\theta}{s_\theta} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_\theta^2}{s_\theta^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \\ \delta Z_L &= \Sigma_L^b(m_b^2) + m_b^2 [\Sigma_L^{b'}(m_b^2) + \Sigma_R^{b'}(m_b^2) + 2\Sigma_S^{b'}(m_b^2)], \end{aligned} \quad (27)$$

where $\Sigma^{\gamma, W, Z, \dots}(k^2)$ are the real parts of the various (transverse components of the) gauge boson self-energy functions⁸. The gauge boson mass counterterms

$$\delta M_W^2 = -Re \Sigma^W(k^2 = M_W^2), \quad \delta M_Z^2 = -Re \Sigma^Z(k^2 = M_Z^2) \quad (28)$$

⁷See also the alternative, though fully equivalent, calculation of ref.[25] within the framework of the “complete” (matrix) α -scheme of ref.[24].

⁸Our self-energy functions [21] are opposite in sign to those of ref.[23].

are enforced by the on-shell renormalization conditions. Moreover, we have decomposed the (real part of the) bottom quark self-energy according to

$$\Sigma^b(p) = \Sigma_L^b(p^2) \not{p} P_L + \Sigma_R^b(p^2) \not{p} P_R + m_b \Sigma_S^b(p^2), \quad (29)$$

and used the notation $\Sigma'(p) \equiv \partial \Sigma(p) / \partial p^2$. Notice that in the minimal α -scheme of ref.[23], where a single renormalization constant is assigned to the quark doublet (t, b) , it is impossible to arrange for the residues of the top and bottom quark propagators to be simultaneously equal to one. In our case only the bottom quark propagator is normalized this way. Consequently, one is forced to introduce a finite wave-function renormalization for the top quark external line (Cf. Fig.2): $\frac{1}{2} \hat{\Pi}_t(m_t^2)$, where

$$\begin{aligned} \hat{\Pi}_t(m_t^2) &= \Pi_t(m_t^2) + \delta Z_L \\ \Pi_t(m_t^2) &= \Sigma_L^t(m_t^2) + m_t^2 [\Sigma_L^{t'}(m_t^2) \not{p} P_L + \Sigma_R^{t'}(m_t^2) \not{p} P_R + 2m_t \Sigma_S^{t'}(m_t^2)]. \end{aligned} \quad (30)$$

Similarly, $\frac{1}{2} \hat{\Sigma}'_W(M_W^2)$ gives the finite wave-function renormalization of the external W (Cf. Fig.3), where the renormalized W -self-energy is given by

$$\hat{\Sigma}_W(k^2) = \Sigma_W(k^2) - \delta M_W^2 + \delta Z_2^W(k^2 - M_W^2) \quad (31)$$

and so

$$\hat{\Sigma}'_W(M_W^2) = \Sigma'_W(M_W^2) + \delta Z_2^W. \quad (32)$$

Putting things together, the general structure of the (minimally) on-shell renormalized $t b W$ -vertex is

$$\begin{aligned} \Gamma_\mu &= \Gamma_\mu^{(0)} + \delta \Gamma_\mu \\ \delta \Gamma_\mu &= i \frac{g}{\sqrt{2}} \gamma_\mu P_L [\delta Z_L + \delta Z_1^W - \delta Z_2^W + \frac{1}{2} \hat{\Pi}_t(m_t^2) + \frac{1}{2} \hat{\Sigma}'_W(M_W^2)]. \end{aligned} \quad (33)$$

In this expression, the full SUSY pay-off to the combined counterterm $\delta Z_1^W - \delta Z_2^W$ turns out to vanish, for the latter combination is seen from eq.(27) to be proportional to the mixed self-energy function $\Sigma^{\gamma Z}(k^2)$ at $k^2 = 0$, and all the chargino-neutralino and sfermion contributions to this function identically vanish at zero frequency [21].

We are now ready to compute the 48 one-loop SUSY contributions from the diagrams of Fig.1 to the vertex form factors. The computation is rather involved, since we retain exact dependence on all masses and keep track of all matrix coupling constants for all SUSY particles in their respective mass-eigenstate bases. Nevertheless, we have managed to present the final analytical results in a fairly compact form. Apart from the tree-level diagram v0 in Fig.1, there are three basic one-loop vertex diagrams: v1, v2 and v3, and all of them are summed over all “ino” and sfermion indices according to the notation of sect.2, which we shall use extensively hereafter.

Diagram v1: Define the following matrices ⁹

$$A_{\pm} \equiv A_{\pm ai}^{(t)} = A_{ai}^{(t)} \pm B_{ai}^{(t)}, \quad A_{\pm}^{(0)} \equiv A_{\pm a\alpha}^{(t)} = A_{a\alpha}^{(t)} \pm B_{a\alpha}^{(t)} \quad (34)$$

and construct (omitting all indices for simplicity) the combinations

$$\begin{aligned} \mathbf{A}^{(1)} &= A_+^* C^R A_-^{(0)} & \mathbf{E}^{(1)} &= A_-^* C^R A_-^{(0)} \\ \mathbf{B}^{(1)} &= A_+^* C^R A_+^{(0)} & \mathbf{F}^{(1)} &= A_-^* C^R A_+^{(0)} \\ \mathbf{C}^{(1)} &= A_+^* C^L A_-^{(0)} & \mathbf{G}^{(1)} &= A_-^* C^L A_-^{(0)} \\ \mathbf{D}^{(1)} &= A_+^* C^L A_+^{(0)} & \mathbf{H}^{(1)} &= A_-^* C^L A_+^{(0)}. \end{aligned} \quad (35)$$

Then the contribution from diagram v1 to the form factors in eq.(24) is the following:

$$\begin{aligned} F_L^{(v1)} &= -\frac{ig^2}{4} [-\mathbf{D}^{(1)} \tilde{C}_0 + M_W^2 \mathbf{D}^{(1)} (C_0 + C_{11} - C_{12}) + m_t^2 \mathbf{D}^{(1)} (-C_0 - 2C_{11} + 2C_{12}) \\ &\quad + m_b^2 \mathbf{D}^{(1)} (-C_0 - C_{11}) + m_t m_b \mathbf{E}^{(1)} (-C_0 - C_{11} + 2C_{12}) \\ &\quad + m_t M_i \mathbf{A}^{(1)} (C_0 + C_{11} - C_{12}) + m_t M_\alpha^0 \mathbf{C}^{(1)} (-C_{11} + C_{12}) - m_b M_i \mathbf{H}^{(1)} C_{12} \\ &\quad + m_b M_\alpha^0 \mathbf{F}^{(1)} (C_0 + C_{12}) + M_i M_\alpha^0 \mathbf{B}^{(1)} C_0 + 2\mathbf{D}^{(1)} C_{24}], \\ F_R^{(v1)} &= -\frac{ig^2}{4} [-\mathbf{E}^{(1)} \tilde{C}_0 + M_W^2 \mathbf{E}^{(1)} (C_0 + C_{11}) + m_t^2 \mathbf{E}^{(1)} (-C_0 - 2C_{11} + C_{12}) \\ &\quad + m_b^2 \mathbf{E}^{(1)} (-C_0 - C_{11} - C_{12}) + m_t m_b \mathbf{D}^{(1)} (-C_0 - C_{11}) \\ &\quad + m_t M_i \mathbf{H}^{(1)} (C_0 + C_{11} - C_{12}) + m_t M_\alpha^0 \mathbf{F}^{(1)} (-C_{11} + C_{12}) - m_b M_i \mathbf{A}^{(1)} C_{12} \\ &\quad + m_b M_\alpha^0 \mathbf{C}^{(1)} (C_0 + C_{12}) + M_i M_\alpha^0 \mathbf{G}^{(1)} C_0 + 2\mathbf{E}^{(1)} C_{24}], \\ H_L^{(v1)} &= -\frac{ig^2 M_W}{2} [m_t \mathbf{E}^{(1)} (2C_0 + 3C_{11} - 3C_{12} + C_{21} - C_{23}) + m_b \mathbf{D}^{(1)} (C_{12} + C_{23}) \\ &\quad + M_i \mathbf{H}^{(1)} C_{12} + M_\alpha^0 \mathbf{F}^{(1)} (C_{11} - C_{12})], \\ H_R^{(v1)} &= -\frac{ig^2 M_W}{2} [m_t \mathbf{D}^{(1)} (2C_0 + 3C_{11} - 3C_{12} + C_{21} - C_{23}) + m_b \mathbf{E}^{(1)} (C_{12} + C_{23}) \\ &\quad + M_i \mathbf{A}^{(1)} C_{12} + M_\alpha^0 \mathbf{C}^{(1)} (C_{11} - C_{12})]. \end{aligned} \quad (36)$$

In the previous expressions we have used (and also explicitly checked) the 3-point function notation from ref.[26], which is an adaptation to the $g_{\mu\nu} = (-, +, +, +)$ metric of the standard formulae from refs.[27, 28]. For diagram v1, all the 3-point functions (C 's and \tilde{C}_0) in (36) have the arguments

$$C = C(p^2, p'^2, m_a(\tilde{t}), M_\alpha^0, M_i). \quad (37)$$

Diagram v2: Define the following matrices:

$$A_{\pm}^{(b)} \equiv A_{\pm a\alpha}^{(b)} = A_{a\alpha}^{(b)} \pm B_{a\alpha}^{(b)}, \quad A_{\pm}^{(t)} \equiv A_{\pm a\alpha}^{(t)} = A_{a\alpha}^{(t)} \pm B_{a\alpha}^{(t)} \quad (38)$$

⁹Matrix indices are always positioned as lower indices, whereas parenthetical upper indices are reserved to denote either flavor (f)=(b),(t) or, in case of (0), to refer to neutralino coupling matrices, when necessary.

and form the combinations

$$\begin{aligned}\mathbf{A}^{(2)} &= R_{b1}^{(t)*} R_{a1}^{(b)} A_+^{(b)*} A_-^{(t)} & \mathbf{C}^{(2)} &= R_{b1}^{(t)*} R_{a1}^{(b)} A_-^{(b)*} A_-^{(t)} \\ \mathbf{B}^{(2)} &= R_{b1}^{(t)*} R_{a1}^{(b)} A_+^{(b)*} A_+^{(t)} & \mathbf{D}^{(2)} &= R_{b1}^{(t)*} R_{a1}^{(b)} A_-^{(b)*} A_+^{(t)}.\end{aligned}\quad (39)$$

The contributions from v2 to the form factors are:

$$\begin{aligned}F_L^{(v2)} &= \frac{ig^2}{4} \mathbf{B}^{(2)} C_{24}, \\ F_R^{(v2)} &= \frac{ig^2}{4} \mathbf{C}^{(2)} C_{24}, \\ H_L^{(v2)} &= \frac{-ig^2 M_W}{4} [m_t \mathbf{C}^{(2)} (-C_{11} + C_{12} - C_{21} + C_{23}) + m_b \mathbf{B}^{(2)} (-C_{12} - C_{23}) \\ &\quad + M_\alpha^0 \mathbf{D}^{(2)} (C_0 + C_{11})], \\ H_R^{(v2)} &= \frac{-ig^2 M_W}{4} [m_t \mathbf{B}^{(2)} (-C_{11} + C_{12} - C_{21} + C_{23}) + m_b \mathbf{C}^{(2)} (-C_{12} - C_{23}) \\ &\quad + M_\alpha^0 \mathbf{A}^{(2)} (C_0 + C_{11})].\end{aligned}\quad (40)$$

In this case the 3-point functions in eq.(40) have the arguments

$$C = C(p^2, p'^2, M_\alpha^0, m_b(\tilde{t}), m_a(\tilde{b})). \quad (41)$$

Diagram v3: The structure of the various contributions is very similar to those from diagram v1. They can be obtained by just replacing $M_i \leftrightarrow M_\alpha^0$ everywhere in eq.(36) and at the same time substituting the set of matrices

$$A_\pm \equiv A_{\pm ai}^{(b)} = A_{ai}^{(b)} \pm B_{ai}^{(b)}, \quad A_\pm^{(0)} \equiv A_{\pm a\alpha}^{(b)} = A_{a\alpha}^{(b)} \pm B_{a\alpha}^{(b)} \quad (42)$$

and

$$\begin{aligned}\mathbf{A}^{(3)} &= A_+^{(0)*} C^L A_+ & \mathbf{E}^{(3)} &= A_-^{(0)*} C^L A_+ \\ \mathbf{B}^{(3)} &= A_+^{(0)*} C^L A_- & \mathbf{F}^{(3)} &= A_-^{(0)*} C^L A_- \\ \mathbf{C}^{(3)} &= A_+^{(0)*} C^R A_+ & \mathbf{G}^{(3)} &= A_-^{(0)*} C^R A_+ \\ \mathbf{D}^{(3)} &= A_+^{(0)*} C^R A_- & \mathbf{H}^{(3)} &= A_-^{(0)*} C^R A_-,\end{aligned}\quad (43)$$

respectively for those in eqs.(34) and (35).

The UV-divergences of the formulae (36,40) are cancelled by adding the contribution from the counterterms in eq.(33) generated by wave-function renormalization of the external fermions. These are sketched in Fig.2, where all indices are understood to be summed over. The (real part of the) self-energy diagram s1 in Fig.2 is given by $-i\Sigma_{s1}^{(b)}(p)$, with

$$\begin{aligned}\Sigma_{s1}^{(b)}(p) &= \left(\frac{-ig^2}{8} \right) \{ [|A_{+a\alpha}^{(b)}|^2 \not{p} P_L + |A_{-a\alpha}^{(b)}|^2 \not{p} P_R] B_1(p^2, M_\alpha^0, m_a(\tilde{b})) \\ &\quad - M_\alpha^0 A_{+a\alpha}^{*(b)} A_{-a\alpha}^{(b)} B_0(p^2, m_a(\tilde{b}), M_\alpha^0) \},\end{aligned}\quad (44)$$

where we have used the 2-point function notation $B_{0,1}$ of ref.[26]. From eq.(44), the terms in the decomposition (29) immediately read off. Similarly, $-i\Sigma_{s2}^{(b)}(p)$ from diagram s2 furnishes

$$\begin{aligned}\Sigma_{s2}^{(b)}(p) &= \left(\frac{-ig^2}{4}\right) \{ [|A_{+ai}^{(t)}|^2 \not{p}P_L + |A_{-ai}^{(t)}|^2 \not{p}P_R] B_1(p^2, M_i, m_a(\tilde{t})) \\ &\quad - M_i A_{+ai}^{*(t)} A_{-ai}^{(t)} B_0(p^2, m_a(\tilde{t}), M_i) \} .\end{aligned}\quad (45)$$

As for the diagrams s3 and s4, the contribution from the former follows from eq.(44) upon replacing $A_{\pm a\alpha}^{(b)} \rightarrow A_{\pm a\alpha}^{(t)}$ and $m_a(\tilde{b}) \rightarrow m_a(\tilde{t})$, whereas the yield from the latter drops from eq.(45) after $A_{+ai}^{(t)} \rightarrow A_{+ai}^{(b)}$ and $m_a(\tilde{t}) \rightarrow m_a(\tilde{b})$. Concerning the SUSY contributions to the external W -self-energy, they are shown in Fig.3. We omit the lengthy analytical expressions, which can be found in ref.[21]. The same reference also quotes the complete SUSY contributions to the self-energies of the Z and of the photon. We have explicitly checked that when putting everything together, UV-divergences cancel in eq.(33) and dimensionful logarithms rescale appropriately. Essential for this are the unitarity of the diagonalizing matrices U, V, N and $R^{(q)}$ from which all coupling matrices have been built up.

With all the one-loop SUSY contributions to the form factors identified, the radiatively corrected amplitude for the process $t \rightarrow W^+ b$ can be written as follows (ϵ^μ being the polarization 4-vector of the W^+):

$$\begin{aligned}\bar{u}(p_b)\Gamma_\mu u(p_t)\epsilon^\mu &= i\frac{g}{\sqrt{2}} \{ [1 + F_L + \delta Z_L + \frac{1}{2}\hat{\Pi}_t(m_t^2) + \frac{1}{2}\hat{\Sigma}'_W(M_W^2)]\mathcal{M}_0 + F_R\mathcal{M}_1 \\ &\quad + H_L\mathcal{M}_2 + H_R\mathcal{M}_3 \} ,\end{aligned}\quad (46)$$

where the structure of the reduced matrix elements $\mathcal{M}_{0,1,2,3}$ should be apparent by comparison of eqs.(33) and (46). The corrected width now follows after computing the interference between the tree-level amplitude and the one-loop amplitude. On the whole we have

$$\begin{aligned}\Gamma &= \Gamma_0(\alpha) \{ 1 + 2F_L + 2\delta Z_L + \hat{\Pi}_t(m_t^2) + \hat{\Sigma}'_W(M_W^2) \\ &\quad + 2\frac{G_1}{G_0}F_R + 2\frac{G_2}{G_0}H_L + 2\frac{G_3}{G_0}H_R \} \equiv \Gamma_0(\alpha)(1 + \delta^{SUSY}(\alpha)),\end{aligned}\quad (47)$$

where

$$\Gamma_0(\alpha) = \left(\frac{\alpha}{s_\theta^2}\right) m_t |V_{tb}|^2 \frac{G_0 \lambda^{1/2}(m_t, M_W, m_b)}{16m_t^4}, \quad (48)$$

with

$$\lambda^{1/2}(x, y, z) = \sqrt{[x^2 - (y+z)^2][x^2 - (y-z)^2]}, \quad (49)$$

is the tree level width, and the polarization sums

$$\begin{aligned}
G_0 &\equiv \sum_{pol} |\mathcal{M}_0|^2 = m_t^2 + m_b^2 - 2M_W^2 + \frac{(m_t^2 - m_b^2)^2}{M_W^2}, \\
G_1 &\equiv \sum_{pol} \mathcal{M}_0 \mathcal{M}_1^* = -6m_t m_b, \\
G_2 &\equiv \sum_{pol} \mathcal{M}_0 \mathcal{M}_2^* = -\frac{m_t}{M_W} \left[m_t^2 + m_b^2 - \frac{1}{2} M_W^2 - \frac{(m_t^2 - m_b^2)^2}{2M_W^2} \right], \\
G_3 &\equiv \sum_{pol} \mathcal{M}_0 \mathcal{M}_3^* = \frac{m_b}{m_t} G_2.
\end{aligned} \tag{50}$$

In eq.(47), $\delta^{SUSY}(\alpha)$ stands for the “relative SUSY correction” in the α -scheme, i.e. the one-loop SUSY correction to the top quark width with respect to the tree-level width, $\Gamma_0(\alpha)$, in that scheme. We emphasize that eq.(48) can also be conveniently parametrized in terms of G_F (Fermi’s constant in μ -decay) by using

$$\frac{\alpha}{s_\theta^2} = \frac{\sqrt{2} G_F M_W^2}{\pi} (1 - \Delta r^{MSSM}), \tag{51}$$

where s_θ^2 is given in eq.(11), with the understanding that M_W and M_Z are the physical masses of the weak gauge bosons. Δr^{MSSM} involves all possible radiative corrections, universal (U) and non-universal (NU) to μ -decay in the MSSM, in particular the “genuine” SUSY ones:

$$\begin{aligned}
\Delta r^{MSSM} &= \Delta r^U + \Delta r^{NU} = -\frac{\hat{\Sigma}_W(0)}{M_W^2} + \Delta r^{NU} \\
&= \Delta r^{SM} + \Delta r^{SUSY},
\end{aligned} \tag{52}$$

where $\hat{\Sigma}_W(0)$, the renormalized self-energy of the W at zero frequency, is obtained from eq.(31). In the present context, Δr^{SM} above includes, apart from conventional SM physics [13], also the contribution from the two-doublet Higgs sector of the MSSM [29]—instead of the single Higgs doublet of the SM—, whilst the “genuine” SUSY part is contained in the second term on the RHS of (52). Clearly, in the new parametrization (G_F, M_W, M_Z) (call it “ G_F -scheme”), the tree-level width of the top quark, $\Gamma_0(G_F)$, is related to eq.(48) through

$$\Gamma_0(\alpha) = \Gamma_0(G_F)(1 - \Delta r), \tag{53}$$

where

$$\Gamma_0(G_F) = \left(\frac{G_F M_W^2}{8\pi\sqrt{2}} \right) m_t |V_{tb}|^2 \frac{G_0 \lambda^{1/2}(m_t, M_W, m_b)}{m_t^4}. \tag{54}$$

Hence the “relative SUSY correction” with respect to $\Gamma_0(G_F)$ is no longer $\delta^{SUSY}(\alpha)$ but

$$\delta^{SUSY}(G_F) = \delta^{SUSY}(\alpha) - \Delta r^{SUSY}. \tag{55}$$

The parameters in the α -and- G_F -schemes are related by the fundamental relation (51), in which Δr^{MSSM} plays a crucial role. As for Δr^{SUSY} in the MSSM, a full one-loop numerical analysis including all possible “genuine” SUSY (universal, as well as non-universal)

contributions has recently been considered in ref.[7] on the basis of an adaptation to the α -scheme of the analytical work of ref.[21], which was carried out in a different (low-energy) renormalization scheme. We refrain from writing out the corresponding formulae. These, together with detailed diagrams contributing to Δr^U and Δr^{NU} in eq.(52), are displayed in ref.[21]. We shall explicitly include these results for the complete numerical analysis presented in the next section.

4. Numerical Analysis and Discussion

The relevant quantities in our analysis are the relative supersymmetric corrections $\delta^{SUSY}(\alpha)$ and $\delta^{SUSY}(G_F)$ to the top quark width in the α - and G_F -schemes. It is well known that in some calculations it is useful to replace the former scheme with a modified (constrained) α -scheme based on the parametrization (α, G_F, M_Z) [13]. Nevertheless, if we would use that framework, then M_W in eq.(48) would no longer be an input parameter but a (model-dependent) computable quantity from the constraint eq.(51). For this reason we prefer to stick to the original α -scheme, where M_W remains an input datum. In this respect it is useful to remember that at LEP 200 the W -mass will be measured with a remarkable precision of $\delta M_W = \pm 28(stat.) \pm 24(syst.) MeV$ [30]¹⁰. Moreover, on top of this it is clear that for processes dominated by mass scales of order $G_F^{-1/2}$ –as in our case– it becomes more appropriate to use the G_F -scheme, (G_F, M_W, M_Z) , which is a genuine high energy scheme for electroweak physics. In this parametrization, large radiative corrections are avoided due to important cancellations between $\delta(\alpha)$ and Δr in eq.(55). This is a reflection of the well-known fact that G_F (as extracted from μ -decay) does not run from low-energy up to the electroweak scale, since large logarithms associated to the renormalization group (RG) do not show up.

Although we shall compare in some respects the radiative corrections in the α - and G_F -schemes, we present the bulk of our numerical analysis in the G_F -scheme. The actual corrections can be straightforwardly computed upon making use of the explicit formulae from sect.3 and the analysis of Δr^{SUSY} from ref.[7]. In practice, however, the numerical evaluation of these formulas is technically non-trivial since it requires exact treatment of the various 2 and 3-point functions for nonvanishing masses and external momenta¹¹. In

¹⁰In ref.[7] it is shown that a measurement of the W -mass with that precision, or even a factor of two worse, would enable us to hint at virtual SUSY effects even if the full supersymmetric spectrum lies in the vicinity of the inaccessible LEP 200 range ($\gtrsim 100 GeV$).

¹¹Leading order calculations performed in the limit of m_t larger than any other mass scale in the decay process have proven to fail in previous analyses within the context of the SM [31, 15].

particular, the exact evaluation of each scalar 3-point function C_0 involves a cumbersome representation in terms of twelve (complex) Spence functions. We refer the reader to the standard techniques in the literature [26, 27, 28] and go directly to present and discuss the final numerical results. They are displayed in Figs.4-8. Apart from the basic input parameters α and G_F , we have fixed [32]

$$M_Z = 91.187 \text{ GeV} , \quad m_b = 4.7 \text{ GeV} , \quad V_{tb} = 0.999 . \quad (56)$$

As for the sparticle masses, they have been required to respect the current phenomenological bounds. On general grounds, the model-independent bounds from $Spp\bar{S}$ and LEP are the following [33]

$$m_{\tilde{l}^\pm} \geq 45 \text{ GeV} , \quad m_{\tilde{\nu}} \geq 42 \text{ GeV} , \quad M(\Psi^\pm) \geq 47 \text{ GeV} , \quad M(\Psi^0) \geq 20 \text{ GeV} . \quad (57)$$

Concerning squarks, the absolute LEP limits are, in principle, similar to those for sleptons [34]. On the other hand the $Spp\bar{S}$ searches for squarks and gluinos (\tilde{g}) amount to a more stringent bound of $m_{\tilde{q}} \geq 74 \text{ GeV}$ for $m_{\tilde{g}}$ around 80 GeV [33]. All the same, this limit becomes poorer as soon as gluinos become heavier. A similar situation occurs for the Tevatron limits, which improve the squark mass lower bound up to $m_{\tilde{q}} \geq 130 \text{ GeV}$ for $m_{\tilde{q}} \leq m_{\tilde{g}} \leq 400 \text{ GeV}$, but if one permits the gluino masses to go beyond 400 GeV the squark mass limit disappears [33]. We shall not consider this extreme possibility. Nevertheless since we want to maximize the possible effects from squarks, we will commence on assuming a mixed mass scenario in which the following limit on squark masses of the first two generations apply:

$$m_{\tilde{q}} \geq 130 \text{ GeV} \quad (\tilde{q} = \tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}) , \quad (58)$$

while we shall explore sbottom and stop squarks with masses starting lower limits

$$m_{\tilde{q}} \geq 75 \text{ GeV} \quad (\tilde{q} = \tilde{t}, \tilde{b}) . \quad (59)$$

We will eventually increase this limit up to the typical bound (58) ascribed to the other families. The reason to single out the third generation of squarks is because the effects of L-R mixing in the mass matrices (specially for the stop, but also for the sbottom) could substantially lower one of the mass eigenvalues (see later on). Furthermore, the squarks of the third generation are those directly involved in the top decay diagrams in Figs.1-2, while the first two generations of squarks only enter through the universal (so-called *oblique* [35]) type corrections shown in Fig.3 –and corresponding ones for the photon and the Z -boson. The numerical analysis shows that these universal contributions to Γ_t , which are generated by the term $\hat{\Sigma}'_W(M_W^2)$ in eq.(47), are rather insensitive to whether we consider the bound

(58) or the bound (59) for all sfermions. It should also be pointed out, in connection to what has been stated above, that the quantity δZ_2^W (see eq.(27)), which is sensitive to the RG-running of α , as well as to the mass splitting among the $T^3 = \pm 1/2$ components in any given $SU(2)_L$ doublet, turns out to cancel from $\delta^{SUSY}(G_F)$, due to the difference between $\delta(\alpha)$ and Δr in eq.(55)¹². Therefore, in the G_F -scheme one expects neither leading RG-type corrections nor any significant enhancement from custodial symmetry-breaking contributions induced by large deviations of the ρ -parameter from unity¹³. The only hope lies in the non-oblique radiative corrections caused by enhanced Yukawa couplings of the form (20), and this is precisely what we are after.

From the point of view of model-building, we have generated the pattern of sfermion masses preserving the bounds (57), (58) and (59) by using models with radiatively induced breaking of the $SU(2)_L \times U(1)_Y$ symmetry such as Supergravity inspired models [5]. For sleptons and the first two generations of squarks we have, using the notation of sect.2,

$$m_{\tilde{f}_{L,R}}^2 = m_f^2 + M_{\tilde{f}_{L,R}}^2 \pm \cos 2\beta (T_{L,R}^3 - Q_{\tilde{f}} s_\theta^2) M_Z^2, \quad (60)$$

where $T_{L,R}^3$ and $Q_{\tilde{f}}$ stand, respectively, for the third component of weak isospin and electric charge corresponding to each member of the multiplet and for each “chiral” species $\tilde{f}_{L,R}$ of sfermion. Finally, the parameters $M_{\tilde{f}_{L,R}}$ are soft SUSY-breaking mass terms [5]. The mass splitting between the $T^3 = +1/2$ and the $T^3 = -1/2$ components in each $SU(2)_L$ doublet is independent of $M_{\tilde{f}_L}$

$$m_{\tilde{f}_L(T^3=+1/2)}^2 - m_{\tilde{f}_L(T^3=-1/2)}^2 = M_W^2 \cos 2\beta, \quad (61)$$

where we have neglected the fermion masses squared of the first two generations against the term on the RHS of eq.(61). The situation for the stop-sbottom doublet, however, requires a particular treatment, due to the possibility of large LR-mixing. We assume it to be the case for the stop squark and proceed to probe this effect in terms of the mass parameter M_{LR} in the stop mass matrix, which can be written as follows:

$$\mathcal{M}_t^2 = \begin{pmatrix} M_{\tilde{b}_L}^2 + m_t^2 + \cos 2\beta (\frac{1}{2} - \frac{2}{3} s_\theta^2) M_Z^2 & m_t M_{LR} \\ m_t M_{LR} & M_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3} \cos 2\beta s_\theta^2 M_Z^2 \end{pmatrix}. \quad (62)$$

Here we have used the fact that $SU(2)_L$ -gauge invariance requires $M_{\tilde{t}_L} = M_{\tilde{b}_L}$ and thus the first entry of the matrix can be written in terms of the L-sbottom mass parameter $m_{\tilde{b}_L}$.

¹²Contrary to all light fermions in the SM, the bounds on sfermion masses given above show that virtual effects from squarks and sleptons must decouple from the photon, as it is also the case for the top quark. Thus no leading RG-corrections from SUSY particles are to be expected in the \overline{MS} -scheme, not even in the α -scheme.

¹³As a matter of fact, custodial symmetry in the MSSM cannot be broken by non-decoupling universal effects, whether statical (ρ -parameter) or dynamical (wave-function renormalization of the gauge bosons). In the limit of $M_{SUSY} \rightarrow \infty$, these effects must vanish [36], irrespective of the parametrization.

To illustrate the effect of the mixing it will suffice to choose the soft SUSY-breaking mass $M_{\tilde{t}_R}$ in such a way that the two diagonal entries of $\mathcal{M}_{\tilde{t}}^2$ are equal— the mixing angle is thus fixed at $\pi/4$. In fact, we have checked that our results are not significantly sensitive to large variations of $M_{\tilde{t}_R}$. For the other sfermions we assume that the R-type and L-type species are degenerate in mass. In this way the only two free parameters are $m_{\tilde{b}_L}$ and M_{LR} . For the mixing parameter, however, we have the proviso

$$M_{LR} \leq 3 m_{\tilde{b}_L}, \quad (63)$$

which roughly corresponds to a well-known necessary, though not sufficient, condition to avoid false vacua, i.e. to guarantee that the $SU(3)_c \times U(1)_{em}$ minimum is the deepest one [37]. Finally, we have also imposed the condition that for our choices of the parameters the induced deviations of the ρ -parameter from 1 should satisfy the bound ¹⁴ [40]

$$|\delta\rho| \leq 0.005. \quad (64)$$

We now come to the discussion of our numerical analysis. In Figs.4-7 we fix $M_{LR} = 0$ and postpone the case of nonvanishing stop mixing until Fig.8. As in ref.[17], we restrict ourselves to the following interval of $\tan\beta$:

$$1 \leq \tan\beta \leq 70. \quad (65)$$

In Fig.4 we display contour lines of $\delta^{SUSY}(G_F)$ in the higgsino-gaugino parameter space (μ, M) . Since we want to compare our SUSY maximum results with those of the 2HDM from refs.[17, 25], we have fixed the value of $\tan\beta$ at the upper limit of the interval (65), which roughly corresponds to the perturbative limit¹⁵. For very large ($\geq m_t/m_b$) values of $\tan\beta$, the two Yukawa couplings (20) are in the relation $1 < \lambda_t < \lambda_b$, and therefore they both give sizeable contributions which translate into relatively large (negative) corrections $\delta^{SUSY}(G_F) = -(5 - 10)\%$ on the top quark width ¹⁶. This is in contradistinction to the maximally expected quantum corrections from the Higgs sector of the MSSM, in which even for $\tan\beta = 70$ the Higgs correction is of only 1% [17, 25]. Notice that for large M and μ the chargino-neutralino contributions in Fig.4 die away, as expected from the decoupling theorem [36]. On Tables I and II we may appreciate more closely the numerical

¹⁴For relatively recent detailed discussions on the ρ -parameter in SUSY, see ref. [38]. The bound (64) is based on pure SM physics and it could be somewhat more relaxed in the framework of the MSSM, as it is exemplified in ref.[39].

¹⁵The lower bound in eq.(65) arises from consistency in GUT models. In this respect, there are phenomenological indications [41] and also recent $SO(10)$ unification models that tend to favor large values of $\tan\beta$ [42]. Proton decay, however, gives $\tan\beta \leq 85$ [43].

¹⁶For $\tan\beta < 1$, λ_b is very small, but λ_t can become rather large and in this region of parameter space one may recover $\delta^{SUSY}(G_F) = -(5 - 10)\%$ too. Nevertheless, values of $\tan\beta$ less than one are disfavoured on several accounts both theoretically and phenomenologically [44].

differences between the corrections in the α - and G_F -schemes for a few choices of the SUSY parameters M, μ and $m_{\tilde{f}}$. The corresponding induced value of $\delta\rho^{SUSY}$ is also provided and it is seen to preserve the bound (64). On the other hand, in Fig.5 (i) we may assess the dependence of $\delta^{SUSY}(G_F)$ on $\tan\beta$ in the full range (65) for typical choices of the other parameters. The corresponding (larger) corrections in the α -scheme are seen in Fig.5 (ii).

We remark that in Fig.4 we have also explicitly displayed the singular contour lines corresponding to the threshold singularities that are expected from the derivatives of the renormalized self-energy functions of the top quark and of the W -gauge boson (Cf. eqs. (30) and (32)) in conventional perturbation theory [45]. For a fixed value of $\tan\beta$ and of the sfermion masses, $m_{\tilde{f}}$, the singularities from wave-function renormalization appear for every numerical pair (μ, M) for which the corresponding eigenvalues of the chargino and neutralino mass matrices satisfy one of the following relations

$$m_t = M_i + m_{\tilde{b}} = M_\alpha^0 + m_{\tilde{t}}, \quad M_W = M_i + M_\alpha^0 = m_{\tilde{t}} + m_{\tilde{b}}, \quad (66)$$

Those (fake) singularities are extremely well concentrated around the set of points satisfying eqs. (66). By explicitly plotting the threshold isolines, it becomes patent in Fig.4 that the typical contours $\delta^{SUSY}(G_F) = -(4, 6)\%$ do not cross the troublesome (pseudo-) singularities. Similarly, the prominent spikes standing up in Figs.5-8 correspond to the projection of the threshold effects onto the different SUSY 2-parameter spaces selected in our analysis.

Several techniques have been devised to tackle this problem— which is not new and is also encountered in other contexts and in particular in pure SM physics [46]: One may e.g. resort to appropriate Dyson-resummation of the propagator of the unstable particle (in our case the top quark and the W -boson) so that the derivative of its self-energy appears in the denominator, or alternatively, one may define the mass and width of the unstable particle through the complex pole position of its propagator, thus avoiding explicit wave function renormalization [45]. In practice, however, since such spurious effects are strongly localized and there is no unambiguous recipe to interpolate the perturbative behaviour, we can get rid of them either by removing the immediate neighbourhood around these points from our numerical analysis or simply by explicitly including these narrow domains but not trusting the results from the inside points, where the perturbative expansion of the S -matrix elements of the theory breaks down. The latter procedure is in actuality our own approach.

The mass of the top quark is, together with a large value of $\tan\beta$, another enhancement factor for our radiative corrections. In Fig.6 (i) we show the behaviour of $\delta^{SUSY}(G_F)$ in terms of the top quark mass in the range $100 \leq m_t \leq 200 \text{ GeV}$. After crossing the transient threshold effects, corrections of order -10% can be achieved. In Fig.6 (ii) we

exhibite for comparison the corresponding corrections in the α -scheme. In order to keep an eye on the bound (64), we plot $\delta\rho^{SUSY}$ as a function of the top quark mass in Fig.6 (iii). We see that for the cases under consideration the bound is saturated at $m_t \simeq 170\text{ GeV}$. Finally, in Figs.7-8 we investigate the dependence of the correction on the mass of the sbottom and stop squarks in the G_F -scheme. Remember that in our framework these masses are fixed in terms of the two parameters $m_{\tilde{b}_L}$ and M_{LR} . For $M_{LR} = 0$, Fig.7 shows the behaviour of the correction against $m_{\tilde{b}}$. For large enough values of this parameter (and therefore of the sbottom and stop masses), the correction decreases (decoupling). However, at the boundary values of the central interval $75\text{ GeV} < m_{\tilde{b}_L} < 130\text{ GeV}$ (where two of the singular-removable-spikes show up on the middle) the correction remains fairly the same. Thus we learn that it does not make much difference to consider stop-sbottom masses near the original lower bound (59) or near the more conservative lower bound (58) that we assumed for the other squark families. This conclusion does not change when we switch on the mixing parameter M_{LR} . It is true that for large values of M_{LR} there emerges a light eigenvalue of the stop mass matrix (62), as seen in Fig.8 (i). All in all the bare fact is that due to a partial cancellation among vertices and external self-energies (reminiscent of a Ward identity), in combination to the aforementioned cancellation of δZ_2^W from $\delta^{SUSY}(G_F)$, the possibility of having a large splitting between one light stop and a heavy sbottom does not render any substantial correction to Γ_t . In Fig.8 (ii) we confirm that outside the singular spikes the correction is not very sensitive (typically $\leq 10\%$) to M_{LR} .

In conclusion, there could be relatively large (few to 10 percent) non-oblique electroweak corrections to the top quark width from the “genuine” SUSY part of the MSSM, due to enhanced Yukawa couplings in the gaugino-higgsino sector. This is in contrast to the one-doublet Higgs sector of the SM, and also to the two-doublet Higgs sector of the MSSM, where in comparable conditions the corrections are one order of magnitude smaller. It is also remarkable that the supersymmetric electroweak corrections are of the same (negative) sign and could be of the same order of magnitude than the conventional QCD corrections [47, 14, 15]. However, whereas the latters are almost insensitive to the top quark mass in the wide range $130\text{ GeV} \leq m_t \leq 300\text{ GeV}$, the formers do significantly vary with m_t in the narrow relevant range $150\text{ GeV} \leq m_t \leq 200\text{ GeV}$. On the whole the QCD + SUSY corrections could reduce the top quark width up to about 10–20%. Consequently, a measurable reduction beyond $\simeq 8\%$ (QCD) could be attributed to a “genuine” SUSY effect. The fact that the gaugino-higgsino sector of the MSSM could afford a non-negligible quantum correction to the top quark decay width, in contradistinction to the inappreciable yield from the scalar Higgs sector of the MSSM, can be traced to the highly constrained structure of the Higgs potential as dictated by SUSY.

Two final remarks: i) In the MSSM, the decay $t \rightarrow W^+ b$ is not the only possible decay; there could be additional electroweak decay modes, such as $t \rightarrow H^+ b$ and $t \rightarrow \tilde{t} \Psi_\alpha^0$, and they have been studied in detail [48]. These SUSY modes notwithstanding, the canonical decay channel would always give a large branching ratio. And in the event that the new decay channels would be closed, due to phase space (i.e. for heavy enough charged Higgs and stop), our SUSY corrections to the canonical decay could still remain sizeable (Cf. Fig.7); ii) In this work, we have not addressed the computation of the strong SUSY corrections to the top quark width, since one usually assumes that gluinos are very heavy and therefore give negligible contributions. Nonetheless this conclusion could change dramatically if one takes seriously the possibility that gluinos could be light (few GeV) [49] or relatively light ($\simeq 80 - 90 GeV$) [50, 51]. In those cases a new SUSY channel, $t \rightarrow \tilde{t} \tilde{g}$, could be open and compete with the canonical mode. Alternatively, it could be closed, but the strong SUSY radiative corrections to the canonical decay be rather significant. It would certainly be interesting to investigate the impact of a light (or relatively light) gluino scenario on the top quark width, but this goes beyond the scope of the present work [52].

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Figure Captions

- **Fig.1** Feynman diagrams, up to one-loop order, for the SUSY vertex corrections to the top quark decay process $t \rightarrow W^+ b$. Each one-loop diagram is summed over all possible values of the mass-eigenstate charginos ($\Psi_i^\pm; i = 1, 2$), neutralinos ($\Psi_\alpha^0; \alpha = 1, 2, \dots, 4$), stop and sbottom squarks ($\tilde{b}_a, \tilde{t}_b; a, b = 1, 2$).
- **Fig.2** One-loop SUSY contributions to the external fermion self-energies in the decay process $t \rightarrow W^+ b$. The notation is as in Fig.1.
- **Fig.3** SUSY vacuum polarization effects on the gauge boson W^+ . Chargino-neutralino notation as in Fig.1. All six sfermion families contribute, whether sleptons ($\tilde{f} = \tilde{e}, \dots, \tilde{\tau}; \tilde{f}' = \tilde{\nu}_e, \dots, \tilde{\nu}_\tau$) or squarks ($\tilde{f} = \tilde{u}, \dots, \tilde{t}; \tilde{f}' = \tilde{d}, \dots, \tilde{b}$).
- **Fig.4** Typical contour (dotted) lines $\delta^{SUSY}(G_F) = -(4, 6)\%$ in the higgsino-gaugino (μ, M) -parameter space for $\tan\beta = 70$ and $m_t = 160 \text{ GeV}$. We have fixed $m_{\tilde{\nu}_f} = 50 \text{ GeV}$ and the lower bounds in eqs.(58) and (59) in combination with the model relation eq.(60). The dashed t -and W -lines correspond to the threshold pseudo-singularities associated to the wave-function renormalization of the t -quark and W -boson, respectively. The blank regions delimited by the full lines are phenomenologically excluded by the constraints $M(\Psi^\pm) \geq 47, M(\Psi^0) \geq 20 \text{ GeV}$. Points a, b and c are used to fix (μ, M) in Figs.5-8.
- **Fig.5** Dependence on $\tan\beta$ of (i) $\delta^{SUSY}(G_F)$ and (ii) $\delta^{SUSY}(\alpha)$ for three widely different choices of the higgsino-gaugino parameters (μ, M) : $(-100, 100)$ (curve a), $(-180, 120)$ (curve b) and $(-60, 200)$ (curve c). Remaining parameters as in Fig.4.
- **Fig.6** Dependence on m_t of (i) $\delta^{SUSY}(G_F)$, (ii) $\delta^{SUSY}(\alpha)$ and (iii) $\delta\rho^{SUSY}$, for the same choices of sfermion and higgsino-gaugino parameters, (μ, M) , as in Fig.5. In Fig.6 (iii), the cases $(-180, 120)$ and $(-60, 200)$ are almost indistinguishable and they are both represented by the line $b \simeq c$.
- **Fig.7** The correction $\delta^{SUSY}(G_F)$ as a function of the sbottom mass. All other masses are fixed as in Fig.4. The three curves correspond to values of (μ, M) as in Fig.5.
- **Fig.8** (i) The evolution of the light and heavy stop masses as a function of the mixing parameter M_{LR} ; (ii) Variation of $\delta^{SUSY}(G_F)$ in terms of M_{LR} for the three choices of (μ, M) as in Fig.5.

Table Captions

- **Table I.** Numerical comparison of the radiative corrections $\delta^{SUSY}(G_F)$ and $\delta^{SUSY}(\alpha)$ for a few choices of chargino-neutralino masses around points a, b and c defined in Fig.4. We have fixed the remaining parameters also as in Fig.4; in particular, $m_{\tilde{b}} = 75 \text{ GeV}$. On the first column of the table we include the induced value of $\delta\rho^{SUSY}$. All numbers are given in percent.
- **Table II.** As in Table I, but with $m_{\tilde{b}} = 130 \text{ GeV}$.

$$m_{\tilde{b}} = 75 \text{ GeV}$$

(μ, M)	$\delta\rho^{SUSY}$	$\delta^{SUSY}(\alpha)$	$\delta^{SUSY}(G_F)$
(-180, 110)	0.437	-7.27	-5.10
(-180, 120)	0.436	-5.75	-3.59
(-180, 130)	0.434	-5.16	-3.01
(-100, 90)	0.474	-9.69	-7.21
(-100, 100)	0.469	-9.33	-6.91
(-100, 110)	0.464	-8.66	-6.28
(-60, 190)	0.443	-11.27	-9.05
(-60, 200)	0.439	-11.51	-9.29
(-60, 210)	0.436	-11.57	-9.36

Table I.

$$m_{\tilde{b}} = 130 \text{ GeV}$$

(μ, M)	$\delta\rho^{SUSY}$	$\delta^{SUSY}(\alpha)$	$\delta^{SUSY}(G_F)$
(-180, 110)	0.310	-3.24	-1.65
(-180, 120)	0.309	-3.13	-1.56
(-180, 130)	0.307	-3.05	-1.48
(-100, 90)	0.347	-7.53	-5.63
(-100, 100)	0.342	-6.70	-4.86
(-100, 110)	0.337	-6.21	-4.41
(-60, 190)	0.316	-10.47	-8.83
(-60, 200)	0.312	-9.94	-8.31
(-60, 210)	0.309	-9.55	-7.92

Table II.

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